

# Gas-phase velocity fluctuations in statistically homogeneous fixed particle beds and freely evolving suspensions using particle-resolved direct numerical simulation

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## Abstract

Gas-phase velocity fluctuations in fixed particle beds and freely evolving suspensions are quantified using particle-resolved direct numerical simulation (PR-DNS). The flow regime corresponds to homogeneous gas-solid systems typically encountered in fluidized bed risers, with high solid to gas density ratio and particle diameter being greater than the dissipative length scales. The contribution of turbulent and pseudo-turbulent fluctuations to the level of gas-phase velocity fluctuations is quantified in flow past fixed particle assemblies. The simulations are then extended to freely evolving suspensions with elastic and inelastic collisions. It is found that for the parameter values considered here (solid volume fraction 0.1 and 0.2, particle to gas density ratio 100 and 1000, and coefficient of restitution in the range 0.7-1.0) the level of gas-phase velocity fluctuations in freely evolving suspensions differs by only 10% from the value for a fixed bed at the same solid volume fraction  $\phi$

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and mean slip Reynolds number  $Re_m$ . Quantification of the Reynolds stress indicates that the second moments of the gas-phase velocity fluctuations are anisotropic, corresponding to unidirectional axisymmetric fluctuations. The anisotropy increases with  $Re_m$  to a maximum that occurs in the range  $10 \leq Re_m \leq 40$ , and then decreases. In addition, the anisotropy decreases with increasing solid volume fraction for all cases considered in this study. The Reynolds stress is decomposed into isotropic and deviatoric parts, and their dependence on  $\phi$  and  $Re_m$  is quantified and explained.

*Keywords:* Fluidized bed, Fixed bed, Freely evolving suspension, Pseudo-turbulent, Gas-phase velocity fluctuations, Reynolds stress

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## 1. Introduction

Gas-solid flows are commonly found in industrial applications such as fluidized-bed combustion, fluid catalytic cracking, coal gasification processes, and biomass energy generation (Fan et al., 2004). The interaction of gas-phase velocity fluctuations with solid particles results in flow structures, such as the core annular structure in circulating fluidized beds. In modeling of dilute suspensions based on averaged equations, Louge et al. (1991) and Bolio et al. (1995) have identified both gas-phase velocity fluctuations and particle-particle interaction as key mechanisms that must be accurately modeled in order to capture this phenomenon. In addition, gas-phase velocity fluctuations enhance the heat transfer and mixing of chemical species inside the fluidized bed. This motivates the current study that is focused on understanding the origin and mechanisms responsible for the generation of gas-phase velocity fluctuations in fluidized beds is essential.

In industrial applications of particulate flows, particles are inertial ( $\rho^{(p)}/\rho^{(f)} \sim 1000$ ) with diameter  $d_p$  ranging from 50 to  $500\mu m$ . In these flows, the particles are usually larger than the length scale of dissipative flow motions  $\eta$ . The interaction of large particles with carrier flow generates pseudo-turbulent gas-phase velocity fluctuations due to the mean slip velocity (Tenneti et al., 2012). The mean slip velocity is an important parameter because it strongly influences both the asymmetric pressure distribution around particles and the formation of boundary layers on particle surfaces. The pressure asymmetry results in the formation of wake structures behind the spheres, which are convected downstream by the carrier flow and contribute to gas-phase velocity fluctuations. The mean slip velocity also generates large velocity gradients in particle interstices. These local velocity gradients modify the dissipation of gas-phase velocity fluctuations, which are no longer characterized by the Kolmogorov length scale, unlike in the case of single-phase turbulence. These mechanisms determine the level of gas-phase velocity fluctuations  $k^{(f)}$  in a particle-laden flow. While these phenomena have been studied for the case of sub-Kolmogorov particles ( $d_p < \eta$ ) (see Balachandar and Eaton (2010), and references therein), they have not been comprehensively studied for large particles ( $d_p > \eta$ ). The level of  $k^{(f)}$  consists of both pre-existing turbulent fluctuations in the carrier flow and pseudo-turbulent gas-phase velocity fluctuations due to the presence of particles. Since the mechanisms for the generation and dissipation of these fluctuations are different, it is useful from a modeling viewpoint to distinguish the pseudo-turbulent from the pre-existing turbulent velocity fluctuations in gas-solid suspensions.

Moran and Glicksman (2003) observed experimentally that the presence

of particles in circulating fluidized beds produces high levels of velocity fluctuations at dilute solid volume fractions that is significant compared to gas-phase turbulent velocity fluctuations in the absence of particles. The hot wire anemometer technique used by Moran and Glicksman (2003) cannot distinguish the directional components of the velocity fluctuations. Furthermore, it is an intrusive measurement technique, as is the specially designed pitot probe used by Doig and Roper (1967). On the other hand, non-intrusive methods of laser Doppler anemometry (LDA) (Lee and Durst, 1982; Rogers and Eaton, 1991; Sato et al., 1996), and particle image velocimetry (PIV) (Oakley et al., 1997; Kiger and Pan, 2000) are limited to very dilute systems due to the difficulties with optical access in denser gas-solid systems.

An alternative approach to investigate gas-phase velocity fluctuations in gas-solid flows is to use particle-resolved direct numerical simulation (PR-DNS) that provides detailed spatial and temporal information of the flow field. For gas-solid flows with particle diameter greater than the Kolmogorov length scale, PR-DNS is necessary to resolve all the scales of the carrier flow turbulence and the flow scales introduced by the presence of particles. PR-DNS has been used to study the interaction of decaying isotropic turbulence with a single particle (Bagchi and Balachandar, 2004; Burton and Eaton, 2005) or a collection of particles (Zhang and Prosperetti, 2005), particle-laden turbulent channel flow (Uhlmann, 2008), gas-solid flows with upstream turbulence (Xu and Subramaniam, 2010), as well as statistically homogeneous pseudo-turbulent gas-solid flows with a mean slip velocity (Tenneti et al., 2012).

In the current study we perform PR-DNS of homogeneous gas-solid sys-

tems with a finite mean slip velocity in fixed particle assemblies and freely evolving suspensions. The fixed particle assembly simulations are used to determine the relative contribution of turbulent and pseudo-turbulent gas-phase velocity fluctuations to the steady level of  $k^{(f)}$ . In these simulations the particles are held stationary and a steady flow is established by imposing a mean pressure gradient that corresponds to the desired flow rate. Use of the fixed-bed simulation methodology for gas-solid flows is justified if the configuration of the particles changes very slowly compared to the time it takes to attain mean momentum balance. Xu and Subramaniam (2010) and Tenneti et al. (2012) argued that the timescale over which the particle configuration changes depends on  $Re_T = d_p T^{1/2} / \nu^{(f)}$ , which is the Reynolds number based on the particle fluctuating velocity characterized by the particle granular temperature  $T$ . Since both experiment (Cocco et al., 2010) and PR-DNS (Tenneti et al., 2010) of gas-solid flows show that  $Re_T$  is low for high Stokes number suspensions, fixed-particle assemblies have been used as a good approximation to freely evolving suspensions. However, a direct comparison between fixed and freely evolving suspensions at the same solid volume fraction and Reynolds number has not been reported to the best of our knowledge.

In real particle-laden flows, the particles move and collide freely. Numerical simulations of freely evolving suspensions have been performed elsewhere (Yin and Koch, 2007) to study the sedimentation of particles under gravity in the presence of fluid. In these studies the steady mean flow Reynolds number attains a unique value that depends on problem parameters (gas and particle densities, solid volume fraction, gravitational acceleration, etc.). Therefore,

in sedimenting suspensions it is not possible to simulate arbitrary Reynolds numbers. However, the simulation of freely evolving suspensions in an accelerating frame of reference (Tenneti et al., 2010) enables us to simulate suspensions at arbitrary Reynolds numbers while maintaining other parameters at fixed values. The results are then compared over a wide range of problem parameters with data from fixed particle assemblies to gain insight into the effect of freely moving particles on gas-phase velocity fluctuations. In addition, we verify the validity of the fixed bed approximation to freely evolving suspensions of high Stokes number particles.

Although PR-DNS is an appropriate tool to investigate particle-laden flows with  $d_p > \eta$ , they are prohibitively expensive for device-scale calculations. Device-scale calculations require CFD simulations of multiphase flows (Halvorsen et al., 2003; Sun et al., 2007) that solve averaged equations for mass, momentum and energy with coupling terms corresponding to interphase interactions. The closure of these equations requires modeling of interphase terms and second moments of velocity fluctuations in both phases. In early modeling attempts (Sinclair and Jackson, 1989; Ding and Gidaspow, 1990; Pita and Sundaresan, 1993) gas-phase velocity fluctuations were usually neglected. The main focus was on the interaction of gas and solid phases through their mean velocities, and the effect of fluctuating velocities in the solid phase that contribute to the granular temperature. The neglect of gas-phase velocity fluctuations that were not previously quantified led to considerable uncertainty in the predictive capabilities of models. Experimental evidence of the importance of gas-phase velocity fluctuations resulted in the emergence of a new class of models in which the effect of gas-

phase velocity fluctuations was also considered. Among the first to model the effect of gas-phase velocity fluctuations were the one-equation model of Louge et al. (1991) and the two-equation model of Bolio et al. (1995) for dilute systems of large particles. Later on, more sophisticated models such as the four-equation  $k$ - $\varepsilon$  model of Simonin (1996) were proposed which are valid for a wide range of volume fraction and Reynolds number. These models are basically extended versions of single-phase  $k$ - $\varepsilon$  models modified to be used in gas-solid flow, and their validity in this regime of gas-solid flow has not been tested. However, PR-DNS of canonical problems plays a key role in shedding light on the physics governing gas-solid flows. The PR-DNS findings can then be used to propose accurate models. In this regard, Tenneti et al. (2012) quantified the level of gas-phase velocity fluctuations from PR-DNS over a wide range of  $\phi$  and  $Re_m$  and proposed an eddy viscosity model.

Recent PR-DNS of gas-solid flows (Uhlmann, 2008; Xu and Subramaniam, 2010; Tenneti et al., 2012) indicate that the gas-phase Reynolds stress is strongly anisotropic. In industrial gas-solid flows, the anisotropy of the velocity fluctuations results in anisotropic heat and scalar transport even in statistically homogeneous suspensions. Therefore, the importance of anisotropy motivates us to quantify the level of anisotropy in gas-solid flows extracted from our PR-DNS data. These anisotropy results in conjunction with the model already proposed for gas-phase velocity fluctuations (Tenneti et al., 2012) can be used to develop a model for the gas-phase Reynolds stress in gas-solid flows.

The rest of the paper reads as follows. In Section 2 the numerical method is described. In Section 3 the interaction of initial turbulence and particle

assemblies in homogeneous systems is addressed. In Section 4 the simulations of freely evolving suspensions are presented. In Section 5 the anisotropy of the Reynolds stress is examined, followed by conclusions in Section 6.

## 2. Numerical method

Numerical investigation of gas-solid flow with particles larger than the Kolmogorov length scale requires PR-DNS. Several numerical methods have been developed for PR-DNS of gas-solid suspensions (Johnson and Tezduyar, 1997; Peskin, 2002; Prosperetti and Oguz, 2001). In this study, we employ the Particle-Resolved Uncontaminated Fluid Reconcilable Immersed Boundary Method (PUReIBM) introduced by Tenneti et al. (2010), which has been successfully used to extract computational drag laws (Tenneti et al., 2011), and quantify gas-phase velocity fluctuations (Tenneti et al., 2012).

PUReIBM utilizes a regular Cartesian mesh wherein the particle surface is described by a discrete number of points. No-slip and no-penetration boundary conditions (BC) are implemented at these points by adding a forcing term to the momentum equation and generating a fictitious flow inside each particle such that the relative velocity of the fluid comes to rest with respect to the particle motion. The instantaneous mass and momentum equations that are solved in PUReIBM are

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

and

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{S} = -\frac{1}{\rho^{(f)}} \mathbf{g} + \nu^{(f)} \nabla^2 \mathbf{u} + \frac{1}{\rho^{(f)}} \mathbf{f} - \mathbf{A}_f, \quad (2)$$

respectively, where  $\mathbf{u}$  is the instantaneous velocity,  $\mathbf{S} = \nabla \cdot (\mathbf{u}\mathbf{u})$  is the convective term in conservative form,  $\mathbf{g}$  is the pressure gradient, and  $\mathbf{f}$  is the

immersed boundary (IB) forcing term that accounts for the presence of particles by ensuring the no-slip and no-penetration BCs at the particle-fluid interface. The IB forcing in PUnReIBM is non-zero only inside the solid particle. Thus, the equations in the fluid phase are unmodified and the solution in the fluid phase is not contaminated due to the IB forcing. The simulation of freely evolving suspension in PUnReIBM is carried out in an accelerating frame of reference that moves with the mean velocity of particles. In Eq. 2  $\mathbf{A}_f$  accounts for the acceleration of the frame of reference (Tenneti et al., 2010).

The computation of IB forcing in PUnReIBM is similar to the direct forcing method proposed by Yusof (1996). The IB forcing at the  $(n + 1)^{\text{th}}$  time-step is specified to cancel the remaining terms in the momentum conservation and force the velocity to its desired value  $u_i^d$  at that location:

$$\mathbf{f}^{n+1} = \rho^{(f)} \frac{\mathbf{u}^d - \mathbf{u}^n}{\Delta t} + \rho^{(f)} \mathbf{S}^n + \mathbf{g}^n - \mu^{(f)} \nabla^2 \mathbf{u}^n + \rho^{(f)} \mathbf{A}_f, \quad (3)$$

The governing equations in PUnReIBM are solved by imposing periodic BCs on fluctuating variables at the boundaries of the computational domain. The velocity field is decomposed into a spatially uniform mean flow that is purely time-dependent and a fluctuating velocity field  $\mathbf{u}'$  that is periodic, i.e.,

$$\mathbf{u}(\mathbf{x}, t) = \langle \mathbf{u} \rangle_{\mathcal{V}}(t) + \mathbf{u}'(\mathbf{x}, t), \quad (4)$$

where the volumetric mean velocity

$$\langle \mathbf{u} \rangle_{\mathcal{V}}(t) = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} \mathbf{u}(\mathbf{x}, t) dv, \quad (5)$$

is obtained by averaging the velocity field over the entire computational domain. Similar decompositions can be written for the non-linear term  $\mathbf{S}$ ,

pressure gradient  $\mathbf{g}$ , and immersed boundary forcing  $\mathbf{f}$  terms as well. Substituting the above decompositions in Eqs. 1 and 2, followed by averaging over the entire computational domain yields the volume-averaged mass and momentum conservation equations. Since the volumetric means are independent of spatial location, mean mass conservation is trivially satisfied. The mean momentum balance in the whole domain is

$$\rho^{(f)} \frac{d \langle \mathbf{u} \rangle_{\mathcal{V}}}{dt} = - \langle \mathbf{g} \rangle_{\mathcal{V}} + \langle \mathbf{f} \rangle_{\mathcal{V}} - \rho^{(f)} \mathbf{A}_f, \quad (6)$$

where the volume integrals of convective and diffusive terms are zero because of periodic BCs. The mean IB forcing term  $\langle \mathbf{f} \rangle$  is computed by volume-averaging the IB force specified in Eq. 3 over the region  $\mathcal{V}$ . The mean pressure gradient  $\langle \mathbf{g} \rangle$  is computed such that we obtain the desired flow rate.

Evolution equations for the fluctuating variables are derived by subtracting Eq. 6 from Eq. 2. Due to the periodicity of the fluctuating fields, a pseudo-spectral method is used with Crank-Nicolson scheme for the viscous terms and an Adams-Bashforth scheme for the convective terms. A fractional time-stepping method based on Kim and Moin’s approach (Kim and Moin, 1985) is used to advance the fluctuating velocities in time. The principal advantage of the PReIBM approach is that it enables the use of regular Cartesian grids to solve for flow past arbitrarily shaped moving bodies without the need for costly remeshing. In addition, the regularity of the mesh simplifies the parallelization of PReIBM.

The salient feature of PReIBM that distinguishes it from other IB methods is that the IB forcing in PReIBM is non-zero only inside the solid phase and the fluid-phase is uncontaminated by the IB forcing. Therefore the velocity and pressure in the fluid phase is a solution to the unmodified

Navier-Stokes equation. In addition, the hydrodynamic force experienced by a particle is computed directly from the stress tensor at the particle surface that is obtained from this uncontaminated fluid flow solution. This feature enables us to directly compare the DNS solution with any random-field theory of multiphase flow (Garg et al., 2011; Tenneti et al., 2011).

Garg et al. (2011) and Tenneti et al. (2011) have shown that PURelBM is numerically convergent and accurate for DNS of gas-solid flows. The method is validated through a comprehensive set of tests: (i) flow past an isolated sphere (ii) Stokes flow past SC and FCC arrangements (ranging from dilute to close-packed limit) compared with the boundary-integral method of Zick and Homsy (1982), (iii) Stokes flow past random arrays of monodisperse spheres compared with LBM simulations of van der Hoef et al. (2005), (iv) moderate to high Reynolds numbers ( $Re_m = 300$ ) in SC and FCC arrangements compared with LBM simulations of Hill et al. (2001), and (v) high Reynolds number flow past random arrays of monodisperse spheres with ANSYS-FLUENT CFD package.

### **3. Gas-phase velocity fluctuations in steady flow past fixed particle assemblies**

While the evolution of gas-phase velocity fluctuations due to particles has been studied for zero mean slip (Zhang and Prosperetti, 2005), the mean slip velocity between the phases is non-zero in fluidized beds. PR-DNS of Tenneti et al. (2012) showed the existence of significant gas-phase velocity fluctuations in homogeneous gas-solid systems initialized with a uniform flow under an imposed mean pressure gradient. However, the study did not address the

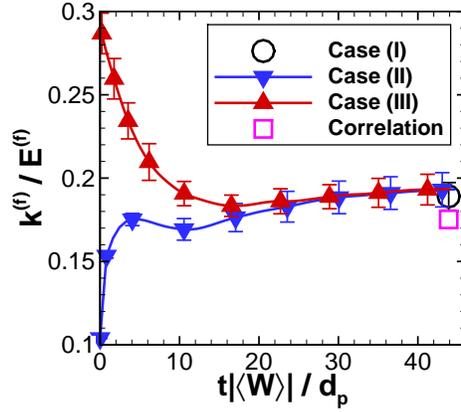


Figure 1: Evolution of  $k^{(f)}$  normalized with mean flow energy. Symbols ( $\nabla$ ) correspond to the simulation initialized with an isotropic turbulence. Symbols ( $\triangle$ ) represent simulation of perturbed velocity field. Symbol ( $\circ$ ) shows the steady value of pseudo-turbulence simulation. Symbol ( $\square$ ) shows the value from the correlation 10.

influence of initial turbulence on  $k^{(f)}$ . Xu and Subramaniam (2010) studied the interaction of a turbulent upstream flow with a fixed particle assembly in an attempt to reproduce the experimental findings of Moran and Glicksman (2003). In their study the upstream flow is initialized with isotropic turbulence and drawn through a uniform configuration of fixed particles by imposing a mean constant pressure gradient. This flow is inhomogeneous in the mean flow direction but gas-phase velocity fluctuations reach a constant value within three particle diameters of entering the bed. The significant result of this study is the enhancement of turbulence as it interacts with solid spheres of the fixed bed.

In the current study, we simulate initially turbulent flow in fixed particle assemblies to quantify the relative magnitude of gas-phase velocity fluctuations arising from turbulent and pseudo-turbulent sources. A uniform distri-

bution of non-overlapping spheres is generated using the Matérn hard-core point process (Stoyan et al., 1995). This is essentially a Poisson point process for particle centers from which overlapping spheres have been removed using an approach called dependent thinning. In our study, the particle configurations correspond exactly to the uniform distribution in Xu and Subramaniam (2010). To address the level of turbulent and pseudo-turbulent gas-phase velocity fluctuations we consider three types of simulations as follows:

1. Case I initialized with a non-turbulent uniform laminar flow
2. Case II initialized with an isotropic turbulent flow
3. Case III initialized with the steady solution of Case I to which the initial isotropic turbulence of Case II is added

The simulations start by imposing a mean pressure gradient along the mean flow such that the Reynolds number reaches the desired value. The Reynolds number based on the mean slip velocity is defined as

$$Re_m = \frac{|\langle \mathbf{W} \rangle| (1 - \phi) d_p}{\nu^{(f)}}, \quad (7)$$

with  $\langle \mathbf{W} \rangle$  being the mean slip velocity. The mean slip Reynolds number is 50 for all cases. Detailed information of flow parameters is provided in Table 1.

The energy in the gas-phase velocity fluctuations  $k^{(f)}$  is computed by ensemble-averaging over multiple independent realizations corresponding to different particle configurations:

$$k^{(f)} = \frac{1}{\mathcal{M}} \sum_{\mu=1}^{\mathcal{M}} k_{\mu}^{(f)}, \quad (8)$$

where the number of realizations  $\mathcal{M}$  is 4 for all cases. The gas-phase fluctuating velocity energy of each realization  $k_\mu^{(f)}$  is

$$k_\mu^{(f)} = \frac{1}{2V^{(f)}} \int_{\mathcal{V}_\mu^{(f)}} u_i''^{(f)} u_i''^{(f)} dV, \quad (9)$$

with  $u_i''^{(f)}$  being the gas-phase velocity fluctuations defined as  $u_i^{(f)} - \langle u_i^{(f)} \rangle$ ,  $\mathcal{V}_\mu^{(f)}$  being the region occupied by gas-phase in  $\mu^{th}$  realization, and  $V^{(f)}$  is the gas-phase volume.

The first verification we perform is comparing the level of pseudo-turbulent gas-phase velocity fluctuations with a correlation to PR-DNS data for slightly higher volume fractions. Using PR-DNS in the range  $0.1 \leq \phi \leq 0.5$  and  $0.01 \leq Re_m \leq 300$ , Tenneti et al. (2012) showed that the energy in pseudo-turbulent velocity fluctuations is characterized by the following correlation

$$\frac{k^{(f)}}{E^{(f)}} = 2\phi + 2.5\phi(1 - \phi)^3 \exp(-\phi Re_m^{1/2}), \quad (10)$$

with  $E^{(f)}$  being the mean flow energy defined as  $\langle \mathbf{W} \rangle \cdot \langle \mathbf{W} \rangle / 2$ . Although there is no simulation data at  $\phi = 0.05$  in that work, Fig. 1 shows that the value obtained from Eq. 10 corresponding to  $\phi = 0.05$  for Case I is 0.18, which is in good agreement (less than 10% difference) with the steady value of  $k^{(f)}$  from our simulation.

Table 1: The numerical and physical parameters of turbulent/pseudo-turbulent simulations.

	$\phi$	$Re_m$	$Re_\lambda$	$d_p/\eta$	$d_p/\Delta x$	$\mathcal{L}/d_p$
Case I: pseudo-turbulent	0.05	50	—	—	20	12.5
Case II: turbulent	0.05	50	12	5	20	12.5
Case III: Case I <sub>s.s.</sub> + iso. turb.	0.05	50	—	—	20	12.5

The initial isotropic turbulence in Case II is generated by the method described by Rogallo (1981) for a Taylor microscale turbulent Reynolds number  $Re_\lambda$  of 12 with the energy spectrum function given by Pope (2000). The corresponding Kolmogorov length scale is selected such that the ratio  $d_p/\eta$  is 5. Fig. 1 also indicates that for Case II, the level of  $k^{(f)}$  starts from the specified initial isotropic turbulence level and increases to the steady state pseudo-turbulent value. The data reveals that the velocity fluctuations arising from the particles in gas-phase are much higher than the turbulent velocity fluctuations for  $Re_\lambda = 12$ , and the principal contribution to the gas-phase velocity fluctuations is the pseudo-turbulent part.

Since in this study, the configurations and simulation parameters are the same as those chosen by Xu and Subramaniam (2010), we can compare the  $k^{(f)}$  between homogeneous simulations of PUnReIBM and inflow/outflow simulations of Xu and Subramaniam (2010). In their study, the flow is inhomogeneous along the flow direction and  $k^{(f)}$  is reported along the mean flow direction. The level of  $k^{(f)}$  increases from the beginning of the bed and represents a 100% increase from the entrance of particle bed and reaches a constant value after an entrance length of  $3d_p$ . Our homogeneous simulation results are similar for Case II where normalized  $k^{(f)}$  starts from an initial value of 0.1 and reaches approximately twice its initial value (0.19) at steady state. This indicates that the enhancement observed by Xu and Subramaniam (2010) is mainly due to the pseudo-turbulent velocity fluctuations, and the steady state value can be characterized by flow parameters  $\phi$  and  $Re_m$  as Eq. 10.

From these results, it is clear that the turbulent velocity fluctuations will

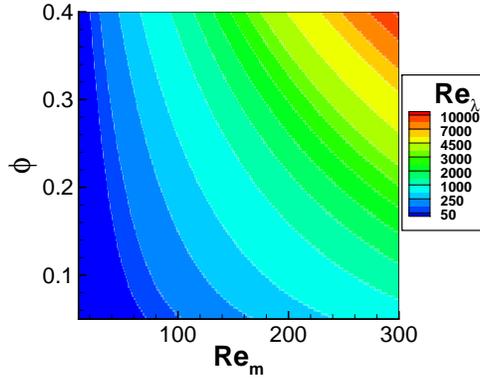


Figure 2: The equivalent Taylor microscale turbulent Reynolds number which generates the same pseudo-turbulent level of velocity fluctuations observed in gas-solid flows. The figure corresponds to  $d_p/\eta = 5$ .

only be important if they are of the same level as, or higher than, the pseudo-turbulent velocity fluctuations. Therefore, it is interesting to determine the Taylor microscale Reynolds number that generates a level of initial turbulence comparable to the steady value of  $k^{(f)}$ , and to ascertain its scaling with  $\phi$ ,  $d_p/\eta$ , and  $Re_m$ . The equivalent  $Re_\lambda$  is determined by using the isotropic turbulence scaling relations in single-phase turbulence (Pope, 2000), and employing the relation between  $Re_L$  and  $Re_\lambda$ , which is  $Re_\lambda = (20Re_L/3)^{1/2}$ . In this context, the turbulent Reynolds number  $Re_L$  is related to the large eddy and dissipative length scales by the expressions  $Re_L = k^{1/2}L/\nu$  and  $Re_L^{-3/4} = \eta/L$ , respectively. The elimination of large eddy length scale  $L$  between these relations, and substitution of single-phase  $k$  by the pseudo-turbulent level of  $k^{(f)}$  leads to an expression for  $Re_L$  as

$$Re_L^{1/2} = \frac{k^{(f)}\eta^2}{\nu^2}. \quad (11)$$

The use of Eq. 10 for the level of  $k^{(f)}$  in above equation and expressing

$Re_L$  in terms of  $Re_\lambda$ , a final expression for the equivalent Taylor microscale turbulent Reynolds number is obtained as

$$Re_\lambda = \sqrt{\frac{20}{3}} \left\{ \phi + 1.25\phi(1 - \phi)^3 \exp(-\phi Re_m^{1/2}) \right\} \left( \frac{\eta}{d_p} \right)^2 \left( \frac{Re_m}{1 - \phi} \right)^2. \quad (12)$$

This expression shows that the equivalent  $Re_\lambda$  is a function of solid volume fraction, mean slip Reynolds number, and particle diameter to Kolmogorov length scale ratio. The dependence of equivalent  $Re_\lambda$  on  $\phi$  is through the expression for the pseudo-turbulent gas-phase velocity fluctuations  $k^{(f)}/E^{(f)}$  in Eq. 10. Since this  $k^{(f)}$  increases with volume fraction (Tenneti et al., 2012), the equivalent  $Re_\lambda$  also increases. In addition, the quadratic dependence of  $Re_\lambda$  on  $Re_m$  indicates that the large eddies associated with the flow structures due to the presence of particles contribute increasingly to the equivalent isotropic  $Re_\lambda$ . The dependency of  $Re_\lambda$  on  $\eta/d_p$  is also represented in Eq. 12 by a quadratic term. To observe the influence of  $d_p/\eta$ , we introduce the definition of the Kolmogorov length scale  $\eta = (\nu^3/\varepsilon)^{1/4}$  to obtain  $d_p/\eta = (\varepsilon d_p^4/\nu^3)^{1/4}$ , which shows that increase of  $d_p/\eta$  causes the dissipation of kinetic energy  $\varepsilon$  to increase dramatically. By maintaining other parameters constant, the large eddy length scale decreases since it is defined as  $L = k^{3/2}/\varepsilon$ . Thus, the decrease of  $L$  results in the attenuation of the turbulent Reynolds number  $Re_L$ , which implies a lower equivalent  $Re_\lambda$ . To gain insight into the behavior of equivalent  $Re_\lambda$ , a contour plot of  $Re_\lambda$  for  $d_p/\eta = 5$  is presented in Fig. 2. This figure reveals that for relatively dense flows at moderate or high mean slip Reynolds numbers, the velocity fluctuations induced by particles are equivalent to high levels of single phase turbulence. Since most PR-DNS of isotropic turbulence are limited to  $Re_\lambda \approx 30$  (Burton and Eaton, 2005; Zhang and Prosperetti, 2005), this indicates that even a moderate value of mean

slip velocity can easily generate high pseudo-turbulent velocity fluctuations compared to the level of turbulent fluctuations (initial or upstream) in the gas phase.

Case III is basically used to examine the relaxation of turbulence in particle-laden flows. The evolution of  $k^{(f)}$  for Case III in Fig. 1 shows that the excess amount of  $k^{(f)}$  decays and the pseudo-turbulent steady state level of  $k^{(f)}$  is recovered. These results showing both attenuation and enhancement of turbulence depending on the initial level of velocity fluctuations indicate that for this choice of  $\phi$  and  $Re_m$ , initial turbulent motions in the gas-phase do not influence the steady value of  $k^{(f)}$ , which corresponds to the pseudo-turbulent fluctuations arising from the interaction of particles with the mean flow. The contour plot in Fig. 2 is useful in quantifying when initial turbulence or the pseudo-turbulent contribution dominates the steady value of  $k^{(f)}$ . Since the mechanism for generation and dissipation of these contributions to  $k^{(f)}$  are different, this provides useful information for the development of predictive multiphase turbulence models in this regime of gas-solid flow. Although the pseudo-turbulent contribution to  $k^{(f)}$  studied in this section is comprehensively quantified by Tenneti et al. (2012), the influence of particle movements on  $k^{(f)}$  is neglected. Thus, in the following section we study the effect of particle motion on the kinetic energy of the carrier flow.

#### 4. Gas-phase velocity fluctuations in freely evolving suspensions

Although fixed beds are good approximations to the particle-laden flows at high particle Stokes number, in reality each sphere moves with an acceler-

ation arising from hydrodynamic and collisional forces. Simulation of freely evolving suspensions enables us to study the effect of physical parameters such as solid to gas density ratio and coefficient of restitution, in addition to solid volume fraction and Reynolds number that are used to characterize fixed bed simulations. In freely evolving suspensions, particles move under the influence of hydrodynamic and collisional forces. In PReIBM the particles are represented in a Lagrangian frame of reference at time  $t$  by  $\{\mathbf{X}^{(i)}(t), \mathbf{V}^{(i)}(t) \mid i = 1 \dots N_p\}$  with  $\mathbf{X}^{(i)}(t)$  and  $\mathbf{V}^{(i)}(t)$  being the position and velocity of  $i^{th}$  particle respectively, and  $N_p$  being the total number of particles. The position and translational velocity of  $i^{th}$  particle evolve according to Newton's second law as

$$\frac{d\mathbf{X}^{(i)}(t)}{dt} = \mathbf{V}^{(i)}(t), \quad (13)$$

$$m \frac{d\mathbf{V}^{(i)}(t)}{dt} = \mathbf{B} + \mathbf{F}_h^{(i)}(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N_p} \mathbf{F}_{ij}^{(c)}(t) \quad (14)$$

where  $\mathbf{B}$  is any external body force,  $\mathbf{F}_h^{(i)}$  is the hydrodynamic force (calculated from the velocity and pressure fields at the particle surface) and  $\mathbf{F}_{ij}^{(c)}$  is the contact force on the  $i^{th}$  particle as a result of collision with  $j^{th}$  particle. Particle-particle interactions are treated using soft-sphere collisions based on a linear spring-dashpot contact mechanics model that was originally proposed by Cundall and Strack (1979) (details are given in Appendix A). The hydrodynamic and contact forces computed at each time step are then used to evolve the position and translational velocity of particles by means of Eqs. 13 and 14, respectively.

The formulation of PReIBM in an accelerating frame of reference en-

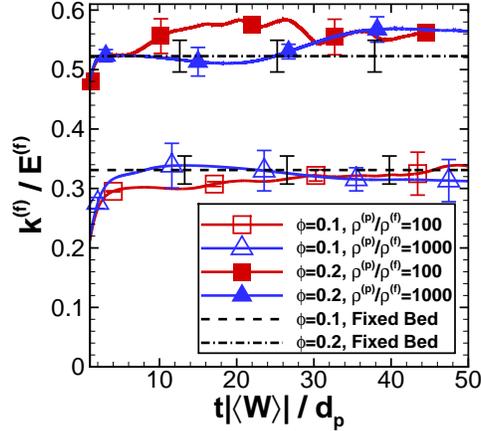


Figure 3: Evolution of  $k^{(f)}$  normalized with mean flow energy for elastic particles with different  $\phi$  and  $\rho^{(p)}/\rho^{(f)}$ .

ables the simulation of not just sedimenting suspensions, but arbitrary mean slip values while maintaining average particle motion at rest in the accelerating frame. In other words, the use of accelerating frame of reference facilitates examining the influence of each flow parameter while maintaining other parameters constant.

In our freely evolving suspension study, all the simulations are performed at  $Re_m = 20$  with two independent realizations for each case. The influence of solid volume fraction is studied by simulating two volume fractions  $\phi = 0.1$  and  $0.2$ . The simulations are initialized with a uniform mean flow for the gas phase and zero granular temperature for the solid phase, and carried out until a steady state solution is obtained in terms of  $k^{(f)}$ . To investigate the effect of particle to gas density ratio we simulated density ratios  $\rho^{(p)}/\rho^{(f)} = 100$  and  $1000$ . The comparison of  $k^{(f)}$  for the two density ratios with elastic particles indicates that the level of gas-phase velocity fluctuations is not influenced

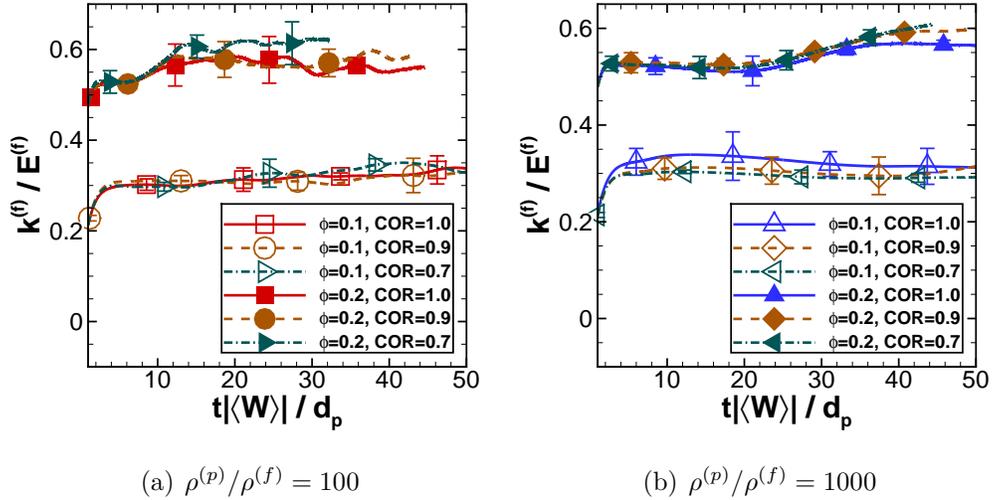


Figure 4: Comparison of  $k^{(f)}$  normalized with mean flow energy among elastic and inelastic cases.

by particle densities significantly as shown in Fig. 3. In this figure  $k^{(f)}$  increases rapidly and then attains a relatively constant value. The evolution of  $k^{(f)}$  is also compared with fixed bed results at the same Reynolds number and solid volume fractions represented by dashed lines in the figure. The difference of  $k^{(f)}$  in freely evolving suspensions from its value in the corresponding fixed bed is less than 10%. This establishes the validity of using fixed bed simulations as an approximation to high Stokes number suspensions undergoing elastic collisions, which was employed in earlier studies (Xu and Subramaniam, 2010; Tenneti et al., 2012).

In freely evolving suspensions with inelastic particle collisions, the energy of the system is dissipated by collisional dissipation among particles as well as the viscous dissipation of the carrier flow. To investigate the effect of the coefficient of restitution (COR) on  $k^{(f)}$ , we selected three values 1.0, 0.9, and 0.7 for this parameter. The comparison of  $k^{(f)}$  for particles with different

CORs, presented in Fig. 4, indicates that  $k^{(f)}$  is not very sensitive to COR and the differences are not statistically significant, especially for the lower volume fraction. In addition, as Figs. 4(a) and 4(b) show, the density ratio has negligible effect on the level of  $k^{(f)}$  for inelastic particles. Similar to the results of elastic particles, the level of  $k^{(f)}$  is in good agreement with those of fixed beds with a maximum difference of 10% for the higher volume fraction. The weak dependence of  $k^{(f)}$  on inelasticity of particles further shows the validity of using fixed beds as an approximation to high Stokes number suspensions.

The independence of  $k^{(f)}$  on COR indicates that these suspensions belong to a regime that is dominated by viscous dissipation as opposed to collisionally dissipative regime (Sangani et al., 1996). To precisely characterize the regime of the gas-solid flows examined in this study, we quantify the ratio of time taken by gas-phase forces to affect particle motions  $t_{\text{fluid}}$  to the average time between collisions  $t_{\text{coll}}$  (Wylie et al., 2003). The fluid timescale is defined as

$$t_{\text{fluid}} = \frac{\sqrt{3T}}{\sigma_A}, \quad (15)$$

where  $\sigma_A$  characterizes the standard deviation in particle accelerations. The mean collisional timescale is computed from the inverse of Enskog collisional frequency (Chapman and Cowling, 1953), that is

$$t_{\text{coll}} = \frac{2}{G\sqrt{T}}, \quad (16)$$

with  $G$  defined as  $8d_p^2n\sqrt{\pi/m}g(d_p)$  in which  $n$  is the particle number density, and  $g(d_p)$  is the radial distribution function of particle assembly at contact.

From Eqs. 15 and 16, the fluid to collisional timescale ratio is

$$\frac{t_{\text{fluid}}}{t_{\text{coll}}} = \frac{\sqrt{3}GT}{2\sigma_A}, \quad (17)$$

which is quantified for the cases simulated in this study, and ranges from 0.19 to 0.9. This confirms that the time required for the gas-phase stresses to significantly influence the particles motion is less than the mean time between the collisions ( $t_{\text{fluid}} < t_{\text{coll}}$ ). In other words, these gas-particle systems are dominated by viscous forces, and the energy loss due to collisional dissipations does not affect significantly the evolution of  $k^{(f)}$ . Thus, the energy of gas-phase fluctuating velocities evolves in a manner similar to fixed bed assemblies.

To better understand the transfer of energy between two phases for freely moving particles, it is useful to investigate the evolution equations of fluctuating energies in both gas and solid phases. This also helps understand the effect of COR through the collisional dissipation. The evolution of  $k^{(f)}$  for a homogeneous suspension is given by (Xu and Subramaniam, 2006; Pai and Subramaniam, 2009; Tenneti et al., 2012)

$$\underbrace{\rho^{(f)}(1-\phi)\frac{d}{dt}k^{(f)}}_{\text{unsteady term}} = \underbrace{\left\langle u_i^{(f)}\tau_{ji}n_j^{(f)}\delta(\mathbf{x}-\mathbf{x}^{(\mathbf{I})}) \right\rangle}_{\Pi^{(f)}} - \underbrace{2\mu^{(f)}\langle I^{(f)}S_{ij}S_{ij} \rangle}_{\varepsilon^{(f)}}, \quad (18)$$

where  $\rho^{(f)}$  is the gas-phase density,  $\boldsymbol{\tau}$  is the stress tensor,  $\mathbf{n}^{(f)}$  is the unit normal vector pointing outward from the gas phase into the solid phase,  $\delta(\mathbf{x}-\mathbf{x}^{(\mathbf{I})})$  is the Dirac delta function representing the gas-solid interface,  $\mu^{(f)}$  is the gas-phase dynamic viscosity, and  $\mathbf{S}$  is the strain rate tensor. The interphase turbulent kinetic energy (TKE) transfer  $\Pi^{(f)}$  in Eq. 18 arises from the fluctuating velocity-stress tensor correlation which is non-zero at gas-solid interface due to the Dirac delta function. The viscous dissipation in the

gas phase  $\varepsilon^{(f)}$  is a sink of energy since  $S_{ij}S_{ij}$  is always a positive quantity (Tenneti et al., 2012). On the other hand, the evolution equation of solid-phase fluctuating kinetic energy  $k^{(p)}$  for a homogeneous system is

$$\underbrace{\rho^{(p)}\phi\frac{d}{dt}k^{(p)}}_{\text{unsteady term}} = \underbrace{\left\langle u_i''^{(p)}\tau_{ji}n_j^{(p)}\delta(\mathbf{x} - \mathbf{x}^{(I)}) \right\rangle}_{\Pi^{(p)}} - \underbrace{\Gamma_{\text{coll}}}_{\text{collisional dissipation}}. \quad (19)$$

In this equation,  $k^{(p)}$  is defined as  $\langle \mathbf{u}''^{(p)} \cdot \mathbf{u}''^{(p)} \rangle / 2$  where  $\mathbf{u}''^{(p)}$  is the solid-phase velocity being  $\mathbf{u}^{(p)} - \langle \mathbf{u} \rangle$ . In addition,  $\rho^{(p)}$  is the solid-phase density, and  $\mathbf{n}^{(p)}$  is the unit normal vector pointing outward from the particle into the gas phase. Note that the unit normal vectors are related to each other as  $\mathbf{n}^{(p)} = -\mathbf{n}^{(f)}$ . It has been shown that the correlation of the fluctuating particle acceleration with the fluctuating particle velocity  $\Pi^{(p)}$  acts as both source and sink of particle kinetic energy (Koch, 1990; Koch and Sangani, 1999; Tenneti et al., 2010).

The kinetic energy of the two-phase mixture  $e_m$  is defined as  $\rho^{(f)}(1 - \phi)k^{(f)} + \rho^{(p)}\phi k^{(p)}$ . The mixture kinetic energy evolution equation is obtained by adding Eqs. 18 and 19 as

$$\frac{de_m}{dt} = \Pi^{(f)} + \Pi^{(p)} - \varepsilon^{(f)} - \Gamma_{\text{coll}}. \quad (20)$$

The principle of conservative interphase TKE transfer introduced by Xu and Subramaniam (2007) implies that the summation of interphase TKE transfer between the two phases ( $\Pi^{(f)} + \Pi^{(p)}$ ) reduces to the inner product of the mean slip  $\langle \mathbf{W} \rangle$  and the mean momentum transfer  $\langle \mathbf{F} \rangle$ , such that

$$\frac{de_m}{dt} = \underbrace{\langle W_i \rangle \langle F_i \rangle}_{\Pi_m} - \varepsilon^{(f)} - \Gamma_{\text{coll}}. \quad (21)$$

For random assemblies, since the mean slip velocity is aligned with the mean interphase momentum transfer (Hill et al., 2001; Tenneti et al., 2010), the resultant interphase TKE transfer  $\Pi_m$  is positive and represents a source of energy, while the viscous dissipation and the collisional dissipation are sinks of energy. At steady state the terms on the right-hand-side of Eq. 21 should balance each other. The  $\Pi_m$  term is computed directly from the DNS data by inner product of the mean slip and the mean drag force measured from the integration of the stress tensor at particle surface. The viscous dissipation is also computed directly from the DNS by the expression given by Tenneti et al. (2012), that is

$$\varepsilon^{(f)} = \frac{1}{V^{(f)}} \int_{\mathcal{V}} I^{(f)} 2\mu S_{ij} S_{ij} dV. \quad (22)$$

The collisional dissipation of the system is estimated from the expression given by Sangani et al. (1996) as

$$\Gamma_{\text{coll}} = \frac{24}{d_p \pi^1 / 2} (1 - e) \rho^{(p)} \phi g(d_p) T^{3/2} \quad (23)$$

where  $e$  is the COR.

The budget analysis of Eq. 21 in Fig. 5 indicates that for these suspensions that start from rest and attain a relatively low steady state granular temperature, the principal balance of terms is between  $\Pi_m$  and  $\varepsilon^{(f)}$ , while  $\Gamma_{\text{coll}}$  is negligible. The balance of energy implies that the level of collisional dissipation in gas-solid suspensions with low granular temperature ( $Re_T/Re_m \approx 0.12$ ) is not significant compared to interphase TKE transfer and viscous dissipation. We also simulated cases with higher initial granular temperature (twice the steady Reynolds number based on granular temperature; results not shown here) and found that although  $\Gamma_{\text{coll}}$  is initially higher than at the steady state,

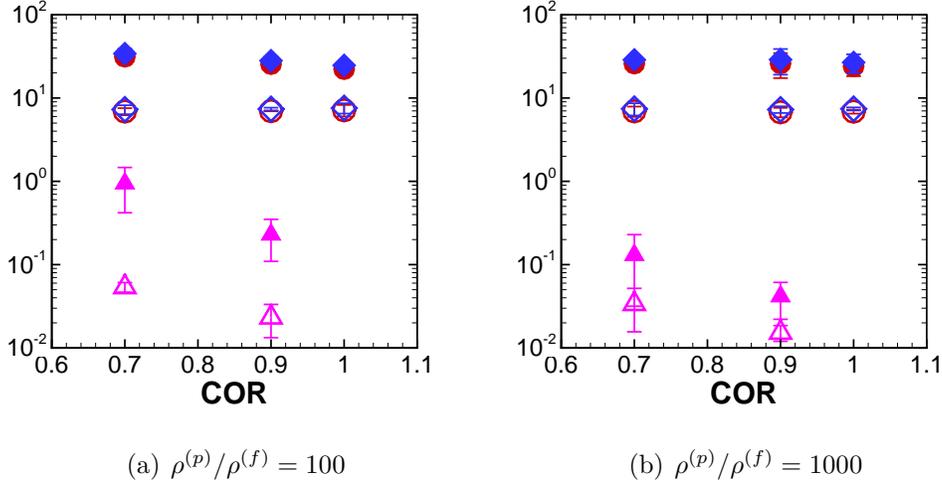


Figure 5: The balance of mixture energy equation terms (Eq. 21) normalized with  $\mu^{(f)}(|\langle \mathbf{W} \rangle| / d_p)^2$ .  $\Pi_m$  ( $\circ$ ),  $\varepsilon^{(f)}$  ( $\diamond$ ),  $\Gamma_{\text{coll}}$  ( $\triangle$ ); hollow symbols ( $\phi = 0.1$ ), filled symbols ( $\phi = 0.2$ ).

it does not affect the steady  $k^{(f)}$  and  $k^{(p)}$ . Thus, the inelasticity of particles does not influence the overall level of energy in the system and the level of  $k^{(f)}$  will be similar for flows with different COR. This finding suggests that the model proposed for fixed beds is applicable to freely evolving suspensions of gas-solid flow as well. In the following section, we quantify the anisotropy of the gas-phase Reynolds stress in fixed beds.

## 5. Reynolds stress tensor of pseudo-turbulent gas-phase velocity fluctuations

The gas-phase Reynolds stress is defined as

$$R_{ij}^{(f)} = \left\langle u_i''^{(f)} u_j''^{(f)} \right\rangle, \quad (24)$$

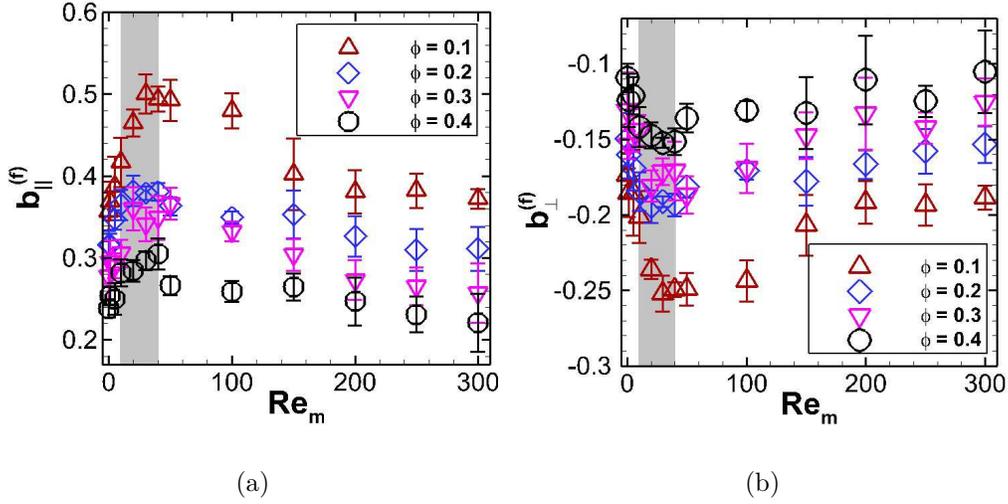


Figure 6: Deviatoric tensor components are shown for different  $\phi$  and  $Re_m$  obtained from fixed bed simulations. The shaded region shows where the maximum of anisotropy occurs.

and its trace represents the level of gas-phase velocity fluctuations  $k^{(f)}$ . The presence of particles introducing anisotropy in the gas-phase Reynolds stress modifies the structure of the carrier phase (Uhlmann, 2008; Xu and Subramaniam, 2010; Tenneti et al., 2012). The gas-phase anisotropy tensor

$$b_{ij}^{(f)} = \frac{R_{ij}^{(f)}}{2k^{(f)}} - \frac{1}{3}\delta_{ij}, \quad (25)$$

is quantified for the fixed-bed particle configurations considered in this study. It is observed in aforementioned studies that flow structures extend in the streamwise direction. Therefore, it is logical to compute the Reynolds stress and the corresponding anisotropy along the streamwise (parallel to mean slip) and spanwise (perpendicular to mean slip) directions which are presented in Figs. 6(a) and 6(b), respectively. The results indicate that the cross-correlation of velocity fluctuations are negligible (not shown here) and the normal component in the parallel direction is dominant compared to the

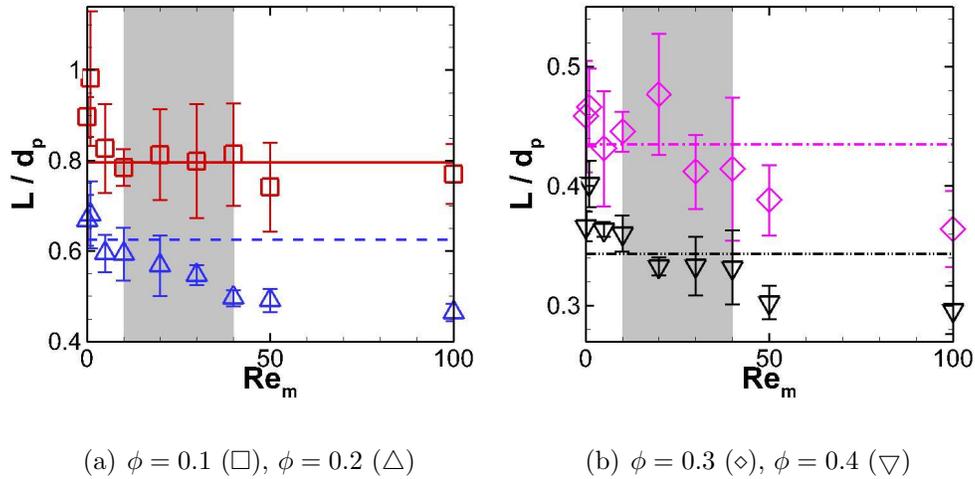


Figure 7: Symbols show length scale of gas-phase velocity fluctuations, while the straight lines show the corresponding local interparticle spacing. The shaded region shows where the maximum of anisotropy occurs.

other normal components in the perpendicular direction. Hence, the state of pseudo-turbulent gas-phase velocity fluctuations is axisymmetric (Pope, 2000).

An accurate model for the Reynolds stress should capture the behavior of anisotropy with flow parameters. Fig. 6 indicates that the level of anisotropy increases with  $Re_m$  from Stokes flow to moderate Reynolds numbers (ranging from 10 to 40) and then smoothly decreases. We hypothesize that this variation in anisotropy is related to the ratio of the length scale associated with gas-phase velocity fluctuations to the local average interparticle spacing. These length scales are quantified using our simulations.

Increase of Reynolds number from low  $Re_m$  flow initiates the separation of boundary layer around particles and a standing eddy forms behind spheres with a characteristic length (Pruppacher et al., 1970) which is a source of

anisotropy in gas-phase Reynolds stress (Yusof, 1996; Uhlmann, 2008). However, defining an appropriate length scale for gas-phase velocity fluctuations in a random particle assembly is not straightforward since the wakes are affected by the presence of neighbor particles. We use the Eulerian two-point correlation of gas-phase velocity fluctuations to define a characteristic length for gas-phase velocity fluctuations  $L_{\parallel}$  as follows:

$$L_{\parallel} = \frac{1}{R_{\parallel}(0)} \int_0^{\infty} R_{\parallel}(r) dr, \quad (26)$$

where  $R_{\parallel}(r)$  is

$$R_{\parallel}(r) = \frac{1}{V^{(f)}} \int_{V^{(f)}} \mathbf{u}_{\parallel}''^{(f)}(\mathbf{x}) \cdot \mathbf{u}_{\parallel}''^{(f)}(\mathbf{x} - \mathbf{r}) dV, \quad (27)$$

the two-point correlation in the parallel direction. This gas-phase velocity fluctuations length scale shown in Fig. 7 decreases with  $Re_m$  due to the fact that the flow structures become finer and less correlated to other surrounding structures. In addition, increase of solid volume fraction makes the gas-phase velocity fluctuations less spatially correlated due to the influence of near particles, and hence the length scale decreases.

We use the radial distribution function  $g(r)$  to define a characteristic local interparticle spacing within a neighborhood of a test particle. Note that other studies used a similar approach to compute the nearest neighbor distance for point particles (Hertz, 1909) and finite size particles (Torquato et al., 1990) in random particle arrangements. The radial distribution function is shown in Fig. 8 for different volume fractions. The radial distribution is the probability of finding a particle at separation  $r$  given that there is a particle at the coordinate origin. By assuming a spherical shell of volume  $4\pi r^2 \delta r$  at

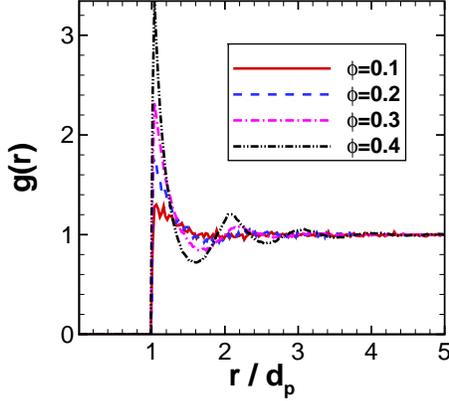


Figure 8: Radial distribution function estimated from 100 realizations for different  $\phi$ .

separation  $r$ , the number of particles in the shell is  $2\pi N_p n g(r) r^2 \delta r$ . Thus, we compute the local interparticle spacing of the suspension using the expression

$$\frac{L_{int}}{d_p} = \frac{1}{d_p} \frac{\int_{d_p}^R g(r) r^2 l f(l) dr}{\int_{d_p}^R g(r) r^2 f(l) dr}, \quad (28)$$

which is a weighted average of distances among particles in a spherical shell between the minimum separation  $d_p$  and  $R$ . In Eq. 28,  $l$  is the surface to surface distance defined as  $r - d_p$ , and  $f(l)$  is a weight function. We choose  $R$  as the second peak of  $g(r)$ , which includes all particles in the local neighborhood of the test particle. The weight function is assumed to have the form  $1/l^p$  with  $p \geq 0$ . Note that the choice of  $p = 0$  causes  $L_{int}$  to be equally weighted by neighbor particles, while choosing higher values for  $p$  preferentially weights the most proximate particles. We select  $p = 1$ , a value that takes into account the importance of all particles in the region between  $d_p$  and  $R$  in a manner where proximate particles contribute more to the local interparticle spacing.

The results indicate that the local interparticle spacing (shown as straight lines in Fig. 7) intersect the length scales of gas-phase velocity fluctuations at the Reynolds numbers where the anisotropy starts to decrease (Fig. 6). At low Reynolds numbers, flow structures formed behind particles elongate with  $Re_m$  that give rise to the increase of anisotropy. After moderate Reynolds numbers ( $10 \leq Re_m \leq 40$ ) the wakes become as large as the gaps among particles and are broken up due to interaction with neighbor spheres. Thus, the anisotropy is characterized by the ratio of gas-phase velocity fluctuations length scale to local interparticle spacing. The break up of elongated structures redistributes the fluctuating velocities energy among Reynolds stress components and decreases the anisotropy.

It is also observed that the increase of solid volume fraction causes an attenuation in the level of anisotropy. Increase of solid volume fraction lessens the local interparticle gaps and does not allow the formation of distinct wake wake structures, leading to the decrease of anisotropy. This is confirmed in Fig. 7 since the length scale of gas-phase velocity fluctuations decreases with volume fraction. Breault et al. (2008) also reported that the axial solid dispersion coefficient of particles in a particle-laden flow decreases with volume fraction, which is interpreted as indication that  $R_{||}$  decreases, while  $R_{\perp}$  increases, and our data is in agreement with this trend observed in experiments.

## 6. Conclusion

In this work we characterized the fluctuations in gas-phase velocity using PR-DNS of steady flow at finite Reynolds number (based on the mean

slip velocity) in fixed particle beds and freely evolving gas-solid suspensions. The evolution of initially isotropic turbulence in a fixed particle bed with mean slip reveals that for the solid volume fraction and Reynolds number considered in this study, the gas-phase velocity fluctuations arising from the presence of particles always relax to their pseudo-turbulent level quantified by Tennesi et al. (2012), irrespective of the level of initial turbulence. The initial level of turbulence required to match the level of pseudo-turbulent velocity fluctuations is estimated to be very high, e.g. at  $Re_m = 100$ ,  $\phi = 0.2$  and  $d_p/\eta = 5$ , the equivalent  $Re_\lambda$  is 320. Therefore, the principal mechanism for producing gas-phase velocity fluctuations in particle-laden flows with  $d_p/\eta > 1$  e.g. fluidized beds, is the interaction of gas and solid phases through the mean slip velocity. Simulations of freely evolving suspensions indicate that  $k^{(f)}$  is a strong function of volume fraction, similar to observations in fixed beds. The change of particle density for high Stokes number particles does not significantly change the level of  $k^{(f)}$  since the time required for the particle configuration to change is longer than the momentum relaxation time. In addition, the variation of inelasticity does not substantially affect  $k^{(f)}$  since viscous dissipation in these systems dominates collisional dissipation. The current study confirms previous findings (Uhlmann, 2008; Xu and Subramaniam, 2010; Tennesi et al., 2012) that the Reynolds stress is highly anisotropic and axisymmetric along the mean slip. Over a wide range of volume fraction ( $0.1 \leq \phi \leq 0.4$ ) the anisotropy increases with Reynolds number as the flow structures form and elongate in the mean slip direction. As  $Re_m$  increases in the range  $10 \leq Re_m \leq 40$ , these structures are affected by neighbor particles that causes them to decorrelate over length

scales smaller than local interparticle spacing, resulting in attenuation of anisotropy. Increase of the volume fraction also decreases the local interparticle gaps and flow structures are broken up leading to attenuation of anisotropy. This work gains insight into the nature of gas-phase velocity fluctuations in gas-solid flows that can be used to propose a model for the Reynolds stress due to the pseudo-turbulent gas-phase velocity fluctuations in homogeneous suspensions.

### **Acknowledgement**

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### **Appendix A. Particle motion equations in PReIBM**

In the soft-sphere approach, the contact mechanics between two overlapping particles is modeled by a system of springs and dashpots in both normal and tangential directions. The spring causes colliding particles to rebound, and the dashpot mimics the dissipation of kinetic energy due to inelastic collisions. The spring stiffness coefficients in the tangential and normal directions are  $k_t$  and  $k_n$ , respectively. Similarly, the dashpot damping coefficients in the tangential and normal directions are  $\eta_t$  and  $\eta_n$ , respectively. The spring stiffness and dashpot damping coefficients are related to the coefficient of restitution and the coefficient of friction (Garg et al., 2010).

The particles considered in this study are assumed to be frictionless. Thus the tangential component of the contact force is zero. Therefore, only normal

component of the contact force  $\mathbf{F}_{n_{ij}}$  is considered at time  $t$  which is given by

$$\mathbf{F}_{n_{ij}} = k_n \delta_{ij} \mathbf{n}_{ij} - \frac{m}{2} \eta_n \mathbf{V}_{n_{ij}}, \quad (\text{A.1})$$

where  $\delta_{ij}$  is the overlap between the particles computed using the relation

$$\delta_{ij} = d_p - | \mathbf{X}^{(i)} - \mathbf{X}^{(j)} |, \quad (\text{A.2})$$

and  $\mathbf{V}_{n_{ij}}$  is the relative velocity in the normal direction that is defined using

$$\mathbf{V}_{n_{ij}} = [(\mathbf{V}^{(i)} - \mathbf{V}^{(j)}) \cdot \hat{\mathbf{r}}_{ij}] \hat{\mathbf{r}}_{ij}. \quad (\text{A.3})$$

The normal vector  $\hat{\mathbf{r}}_{ij}$  is the unit vector along the line of contact pointing from particle  $i$  to particle  $j$ .

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