Engineering and physical challenges to engineered crops

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Why study sugar transport in plants?
Fig. 1: Plant's rørsystemer. Vandet bevæger sig i xylemet fra roden mod bladet, mens sukkeret løber i phloemet i den modsatte retning [1]. Xylemet sidder i midten og udgør langt den største del af stammens areal. Vandtransporten foregår mellem den yderste håndfuld åringe. Phloemet sidder i et meget tyndt lag ligge under barken.

Liquid dynamics in plants
## Liquid dynamics in plants

### Phloem
- Sugar transport
  - Sugar: 1 kg/day
  - Water: 4 kg/day
- Cell diameter: 10 μm
- Flow velocity: 100 μm/s
- Reynolds number: $10^{-3}$

### Xylem
- Water transport
  - Water uptake: 100 kg/day
  - Evaporation: 95 kg/day
  - Photosynthesis: 1 kg/day
  - Phloem: 4 kg/day
- Cell diameter: 100 μm
- Flow velocity: 1 mm/s
- Reynolds number: $10^{-1}$

Physical challenges
Leaf size
Sugar flow

phloem tube

osmotic flow

sugar
The leaf, an osmotic pump

1 cm, 5 frames/hour
Sugar speed – scaling analysis

• **Leaf dominant**
  \[ u = \frac{2L_p l}{r} \Delta p \]
  \[ R_l = \frac{1}{2\pi rlL_p} \]

• **Stem dominant**
  \[ u = \frac{r^2}{8\eta h} \Delta p \]
  \[ R_s = \frac{8\eta h}{\pi r^4} \]

• **Münch number**
  \[ M\dddot u = \frac{\text{STEM}}{\text{LEAF}} = \frac{16L_p \eta l h}{r^3} \]
Engineering challenge #1: Measuring phloem flow speed

- Radioactive tracers
  Minchin and Troughton

- Nuclear magnetic resonance imaging (NMR)

- Fluorescent dye

Savage, Zwieniecki, Holbrook
Plant Physiology 163 (2013)

Sugar speed – scaling analysis

\[ \bar{u} = \frac{L_p l r^2}{r^3 + 8L_p \eta l h} \Delta p \]

\[ u[\mu m/s] \]

\[ r[\mu m] \]

\[ \Delta p = 0.25 MPa \]

\[ \Delta p = 0.10 MPa \]

Limits to Leaf Size

- Energy flux \( E = kcu = \frac{2r^2L_pl}{r^3 + 16L_p\eta lh}kc\Delta p \).

\( E_{\text{max}} = \frac{1}{8} \frac{r^2}{\eta h} kc\Delta p \).
Upper limit to leaf size

- Large leaf, fast flow
  - Cost of maintaining vasculature
  \[ C = \gamma l \pi r^2 \]

\[ E - C = 0 \Rightarrow l_{\text{max}} = \frac{1}{16} \frac{2r^2 L_p k c \Delta p - \gamma r^3}{\gamma L_p \eta} \frac{1}{h} \]

\[ l_{\text{max}} \sim \frac{1}{h} \]
Upper limit to leaf size

- Large leaf, fast flow
  - Stop growth when close to max output

\[ E \sim (1 - \tau)E_{\text{max}} \quad \tau \ll 1 \]

\[ l_{\text{max}} = \frac{1}{16} \frac{r^3}{\tau L_p \eta h} \]

\[ l_{\text{max}} \sim \frac{1}{h} \]
Lower limit to leaf size

- Small leaf, slow flow
  - Bulk flow faster than diffusion

\[ Pe = \frac{uL}{D} \geq 1 \]

Péclet number

- \( L \): Characteristic cell-to-cell distance (10-100 µm)
- \( D \): Diffusivity

\[ l_{\text{min}} = \frac{1}{16} \frac{r^3}{L_p \eta} \frac{1}{(h_{\text{max}} - h)} \]

\[ h_{\text{max}} = \frac{r^2 L \Delta p}{8 \eta D} \frac{1}{Pe} \]
\[ l = l_{\text{max}} \sim \frac{1}{h} \]

\[ l = l_{\text{min}} \sim \frac{1}{h_{\text{max}} - h} \]

(90% of max output)

(Péclet number \( Pe = 1-10 \))
Physical challenges

(a) xylem (water) and phloem (sugar)

(b) Streaming, stomata, and evaporation
Engineering challenge #2: Mapping the vascular architecture

Serial light micrographs

X-Ray Computed tomography

Brodersen et al. New Phytologist 191 (2011)
Lee et al. Microscopy Res. Tech. 76 (2013)
Size of individual phloem tubes

Black locust  Norway spruce  Squash  Bamboo

20 µm

Sugar speed depends on the phloem tube size

\[ u = \frac{2r^2 L_p l}{r^3 + 16\eta L_p l h} \Delta p \]

- Fixed leaf and stem length, speed optimal when \( R_s = R_l \) (\( M \dot{\ddot{u}} = 1 \))

\[ r^3 \sim L_p \eta l h \]
\[ r^3 \sim L_p \eta \lambda h \]

\[ M \ddot{u} = 1 \]

Phloem Sap Composition

• ~20% sugars
  – sucrose, glucose, fructose, sorbitol, mannitol, raffinose, stachyose…

• ~1%
  – Proteins, amino acids, hormones, signaling molecules
Engineering challenge #3: Drawing blood from a plant

- Bleeding
- Aphid stylectomy

Munns (Ed.) Plants in Action (2010)
Zimmerman. Plant Physiology 32 (1957)
Sugar flow in the stem

- **Volume flow**
  \[ Q = \frac{\pi r^4}{8} \frac{\Delta p}{L} \frac{1}{\eta(c)} \]

- **Sugar mass flow** \( J = Qc \)
  \[ J = \left( \frac{\pi r^4}{8} \frac{\Delta p}{L} \right) \frac{c}{\eta(c)} \]
Sugar mass flow

\[ J \sim \frac{C}{\eta} \]

\[ \eta \]

\[ \frac{J}{J_{\text{max}}} \]

\[ c \text{ [\% w/w]} \]
Sugar mass flow

\[ J \sim \frac{c}{\eta} \]

\[ \eta \]

\[ \frac{J}{J_{\text{max}}} \]
Nectar Drinking

Hummingbirds

• Surface tension
  – Drink through cylindrical tube formed by folding tongue

\[ \Delta p = \frac{2\sigma}{a} \]

\[ \ell = \left( \frac{a\sigma t}{2\eta} \right)^{1/2} \]

\[ J \sim \frac{c}{\eta^{1/2}} \]

Sugar uptake

\[ J = c \frac{\pi a^2 \ell(T)}{T + T_0} \]

\[ \pi a^2 \frac{d\ell}{dt} = \frac{\pi a^4}{8\eta l} \Delta p \]
Nectar Drinking

Hummingbirds
- Surface tension
  \[ J \sim \frac{c}{\eta^{1/2}} \]

Bees
- Viscous dipping
  \[ J \sim \frac{c}{\eta^{1/6}} \]
\[ J \sim \frac{C}{\eta^{1/n}} \]
Simple Model for Flow Impeded by Concentration

\[ c^* = \frac{c}{c_{\text{opt}}} \quad \eta^* = \frac{\eta}{\eta(c_{\text{opt}})} \quad J^* = \frac{J}{J(c_{\text{opt}})} \]

\[ \frac{\partial J^*}{\partial c^*} = A - Bc^* \]

\[ J^*(0) = 0 \]

\[ J^*(1) = 1 \]

\[ \left. \frac{\partial J^*}{\partial c^*} \right|_{c^* = 1} = 0 \]
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Data collapse

\[ J^* = c^*(2 - c^*) \]

Traffic flows


http://www.wikipedia.org