Fluctuations, hydrodynamics and scale separation in gas–solid flows

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Fluctuations in the number of particles (or droplets and bubbles) are observed in experiments of multiphase flows, and these also manifest as fluctuations of the dispersed–phase volume fraction. The intensity of these fluctuations is characterized using a mathematical description based on second–order statistics. The intensity and range of fluctuations in the clustering regime of a granular gas is characterized using hard–sphere simulations. The influence of these fluctuations on conservation laws is first investigated through the hydrodynamic equations that arise in the standard average number density representation in kinetic theory of granular and gas-solid flow. The effect of number fluctuations on the unclosed conditional acceleration term in the kinetic theory of gas–solid flow is examined using data from particle–resolved direct numerical simulation. The intensity and range of fluctuations suggest a breakdown of scale separation that underlies continuum descriptions of gas–solid flow, such as the two–fluid theory and the first–order hydrodynamic description that arises from the Boltzmann–Enskog approach to gas-solid flow. Fluctuation hydrodynamics of gas-solid flow is proposed as a route to rigorously incorporate the effect of fluctuations on conservation laws at both the first and second–order levels of description.

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I.  INTRODUCTION

It is clear from visual observation of many multiphase flows that the number of dispersed-phase elements (solid particles, droplets or bubbles), and the geometric volume associated with them, can vary significantly in time and space. In this work we seek to characterize these fluctuations using a mathematical framework that relates them to foundations of the kinetic theory of gas–solid flow and the two–fluid theory.

Fluctuations are closely related to clusters, which is a term often used to describe spatial patterns in particle point fields. Clustering of large inertial particles is experimentally observed in gas–solid flows\textsuperscript{1–5} such as fluidized beds. Clusters have been directly observed in fluidized bed risers using high-speed particle imaging\textsuperscript{6,7}. It is also known from simulations that even smaller inertial particles and droplets preferentially concentrate in turbulent flow\textsuperscript{8–11}.

Clustering arises due to particle-fluid (hydrodynamic) and particle-particle interactions (inelastic collisions, cohesion, electrostatics). It is important because it affects interphase momentum transfer and transport phenomena\textsuperscript{12}. Particle–resolved DNS can quantify these effects\textsuperscript{13,14}. The theoretical significance of clustering, or fluctuations in volume fraction, lies in its implications for the validity of first–order statistical theories of multiphase flow, such as the kinetic theory for gas-solid flow (KTGF) based on the one–particle distribution function, or the Eulerian two–fluid/multifluid theory.

MATHEMATICAL FORMULATION

In a realization of a multiphase flow we encounter a number $N(\mathcal{V})$ of dispersed particles (or drops or bubbles) in a region $\mathcal{V}$ in physical space (see top panel of Fig. 1). In general $N(\mathcal{V})$ is a random number, and the random nature of $N$ is crucial to representing fluctuations in multiphase flows.

The expected (or mean) value of $N(\mathcal{V})$ is denoted $\langle N(\mathcal{V}) \rangle$ (angle brackets denote averaging), and it can be formally defined through probability and measure theory\textsuperscript{15}. The expected value is a well defined quantity independent of any procedure used to estimate it\textsuperscript{16}. Nevertheless it is useful from a conceptual viewpoint to think of $\langle N(\mathcal{V}) \rangle$ as obtained by averaging over an ensemble of statistically identical realizations\textsuperscript{17}. It is easy to show that for a gen-
FIG. 1. The number of particles $N(\mathcal{V})$ in a realization of a multiphase flow in a region $\mathcal{V}$ in physical space is a random variable (top panel). Averaging $N(\mathcal{V})$ over an ensemble of statistically identical realizations we obtain the expected (or mean) value $\langle N(\mathcal{V}) \rangle$. Note that the physical region $\mathcal{V}$ is unchanged, i.e., $\langle N(\mathcal{V}) \rangle$ is not an estimate obtained by volume-averaging.

General multiphase flow that is neither statistically stationary nor statistically homogeneous, ensemble-averaging is the only well defined procedure for estimating averages\textsuperscript{18}.

Under fairly mild conditions detailed in\textsuperscript{15,19}, $\langle N(\mathcal{V}) \rangle$ has a density $n(x)$ in physical space such that

$$
\langle N(\mathcal{V}) \rangle = \int_{\mathcal{V}} n(x) \, dx,
$$

and where $n(x)$ is simply the (average) number density of particles in physical space. This (average) number density forms the basis of the kinetic theory of granular/gas-solid flow (KTGF). On the basis of this number density, a number-averaged particle velocity is defined that leads to the hydrodynamic description in terms of mean mass and momentum balance equations. We call this a first-order description to distinguish it from a second-order description of fluctuations that follows.

II. FIRST-ORDER DESCRIPTION: MEAN FIELD HYDRODYNAMICS

In a mean-field description of a granular gas or gas-solid flow, the number density

$$
n(x, t) = \int_{\mathcal{V}} f(x, v, t) \, dv
$$
can be defined in terms of the Klimontovich density\textsuperscript{20} or one–particle distribution function
\[ f(x, v, t) = \langle f' \rangle = \left( \sum_{i=1}^{N} \delta(v - V^{(i)}(t))\delta(x - X^{(i)}(t)) \right), \]
where \( \{X^{(i)}(t), V^{(i)}(t), i = 1, \ldots N\} \) are the position and velocity of the \( N \) particles in the ensemble\textsuperscript{21}. The number density evolves by
\[ \frac{\partial n}{\partial t} + \vec{\nabla}_x \cdot (u n) = 0, \tag{1} \]
where the mean particle velocity field is defined as
\[ u(x, t) = \frac{1}{n} \int_v v f(x, v, t) dv. \]

The first–order description in terms of average number density is closely related to the two–fluid theory, which is also a first–order description. The exact relationship between the one–particle distribution function approach and the two–fluid theory is detailed by Pai and Subramaniam\textsuperscript{22}. Here the terminology “first–order” is used to denote any theory based on the mean measure of number or volume. Note that one can still have second moments of velocity at the level of a first–order theory: for example, the second moment of particle velocity, the trace of which gives the particle granular temperature. In this terminology, the particle granular temperature is a second moment of first–order. The second–order description that is described in the following section is based on the second moment of the random measure \( N \).

III. SECOND–ORDER DESCRIPTION: FLUCTUATIONS

Fluctuations in number, and the importance of second–order effects, can be assessed through second–moment measures of point fields\textsuperscript{23}. Since \( N(\mathcal{V}) \) is a random measure on a set \( \mathcal{V} \), its expected value \( \langle N \rangle(\mathcal{V}) \) is a mean measure. The second moment of \( N \) is \( \langle N^2(\mathcal{V} \times \mathcal{V}) \rangle \), and is defined on a product space. It is natural to define the variance of \( N \) as
\[ \text{var}(N) = \langle N^2 \rangle - \langle N \rangle^2, \]
and the variance captures the effect of fluctuations in \( N \). An important difference between stochastic point processes and continuous random fields that are encountered in single–phase turbulence is the reference value for the level of fluctuations. In single–phase turbulence the
limit of zero velocity fluctuations is meaningful and corresponds to a nonturbulent (laminar) flow. Or if one considers mixing of a scalar $\phi$ in a turbulent flow, the reference state with zero fluctuations is the well-mixed state where $\text{var}(\phi) = 0$ everywhere in the domain and $\phi = \langle \phi \rangle$. However, for stochastic point processes the reference state is the homogeneous Poisson process that is characterized by complete randomness, for which the value of $\text{var}(N)$ is not zero but given by

$$\text{var}(N) = \langle N \rangle.$$

A. Intensity of fluctuations

The intensity of number fluctuations is characterized by the scaled variance

$$\frac{\text{var}(N)}{\langle N \rangle^2} = \frac{\langle N^2 \rangle}{\langle N \rangle^2} - 1,$$

but often factorial moments are used in stochastic point process theory because the variance has no density (owing to an atomic contribution), whereas the factorial moments do. The second factorial moment measure $\alpha^{(2)}(\mathcal{V}) = \langle N(N - 1) \rangle$ can be defined on the same product space as $\langle N^2 \rangle$ by the following expression

$$\alpha^{(2)}(\mathcal{V} \times \mathcal{V}) = \langle N^2(\mathcal{V} \times \mathcal{V}) \rangle - \langle N(\mathcal{V}) \rangle.$$

(2)

Normalization by $\langle N \rangle^2$ results in the scaled second factorial moment (SSFM)

$$\alpha^{(2)}(\mathcal{V} \times \mathcal{V})/\langle N(\mathcal{V}) \rangle^2,$$

which provides information regarding the spatial distribution of fluctuations in number of particles. If particle centers are distributed according to a homogeneous Poisson process, then the SSFM is unity. Deviations of the SSFM from unity can be used to detect clustering in a particle system$^{24}$.

B. Length scale associated with fluctuations

Although the Poisson process is a useful reference distribution to quantify the intensity of fluctuations, it is not a realistic model for multiphase flows because it allows overlap of finite–sized particles. Essentially its property of complete independence and randomness
FIG. 2. The principal length scales that influence fluctuations in number. The characteristic length scale associated with the measurement region $V$ is denoted $\ell$, while the diameter of the spherical particles that are considered is $d_p$.

do not allow the exclusion of neighbor particles within a hard-core distance of $d_p$ from the center of any particle. A consequence of this property of the Poisson point process is that the intensity of number fluctuations as measured by the SSFM (which is unity) is independent of the size of the measurement region $V$. Figure 2 shows $\ell$, which is a characteristic length scale associated with the measurement region $V$, and $d_p$, which is the diameter of the spherical particles that are considered. For a homogeneous Poisson process, the SSFM is unity for any value of $\ell/d_p$. However, the same is not true of multiphase flows.

Modifications to the Poisson process that remove overlapping points through a procedure called dependent thinning\textsuperscript{25,26} result in the hard-core Matérn process that is a good model for a hard–sphere system of particles at low volume fraction (or density). In the Matérn process the intensity of fluctuations is linked to the length scales of measurement, and depends on $\ell/d_p$: the ratio of characteristic length scale of the measurement volume to the particle size. For $\ell/d_p \gg 1$, the intensity of number fluctuations in the Matérn process tends to that of the Poisson, and becomes independent of the size of the measurement volume. Simulations of packings of hard–sphere systems also reveal the same dependence. This is in accord with our physical intuition that for molecular gases measured on “macroscales”, fluctuations in number are negligible. Indeed, this forms the basis for the well known continuum hypothesis. Figure 3 shows a reproduction of a sketch explaining the validity of the continuum hypothesis adapted from Batchelor’s classic introductory text on fluid dynamics\textsuperscript{27}. In Fig. 3 the abscissa denotes the characteristic length scale of the measurement volume $\ell$ and the
FIG. 3. A sketch showing the validity of the continuum hypothesis based on separation of scales in molecular gases (see e.g., Batchelor[27]). The annotations \( d_p, \ell_{\text{meso}} \) and \( \ell_{\text{macro}} \) denote the micro, meso and macroscale length scales respectively.

Ordinate represents the measured density that we can interpret equivalently as that of number or mass. The annotations showing the particle diameter \( d_p \), and meso– and macroscales, denoted \( \ell_{\text{meso}} \) and \( \ell_{\text{macro}} \), respectively have been added to the original sketch to explain the scales introduced in this work. Note that for \( \ell/d_p \sim 1 \) the intensity of fluctuations in both the Matèrn hard–core process and in hard–sphere packing simulations increases, which is in accord with the sketch in Fig. 3.

The characteristic length scale \( l_g \) beyond which the intensity of fluctuations in both the Matèrn hard–core process and in hard–sphere packing simulations is the same as the Poisson distribution can be inferred from the density of the second factorial moment

\[
\alpha^{(2)}(\mathcal{V} \times \mathcal{V}) = \int_{\mathcal{V}} \int_{\mathcal{V}} \rho^{(2)}(\mathbf{x}_1, \mathbf{x}_2) \, d\mathbf{x}_2 \, d\mathbf{x}_1,
\]

which for statistically homogeneous systems is simply the unnormalized pair–correlation function:

\[
\rho^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = n^2 g(\mathbf{r}),
\]

where \( \mathbf{r} = \mathbf{x}_2 - \mathbf{x}_1 \) is the pair separation vector. The length scale associated with the pair–correlation beyond which the intensity of fluctuations is the same as the Poisson process can be defined as

\[
l_g \equiv \{ r^* : g(r) = 1, \forall r > r^* \}.
\]

Exclusion is of course the most short–ranged form of particle–particle interactions that generates number fluctuations on the scale of the particle diameter \( d_p \). In multiphase flows,
other forms of particle–particle interaction such as inelastic particle collisions and cohesion can result in more long–ranged interactions. Furthermore, particle–fluid interactions can result in long–ranged interactions that create fluctuations in number on length scales greater than $d_p$.

IV. INTENSITY AND RANGE OF FLUCTUATIONS FROM HOMOGENEOUS GRANULAR GAS SIMULATIONS

One of the simplest particle systems that exhibits clustering is the homogeneous cooling granular gas in which particles collide in the absence of any ambient fluid. Since clustering results in fluctuations in number and volume, this is a useful model system in which we can characterize the intensity and range of fluctuations. In a granular gas particles interact through inelastic collisions from a specified initial state, and in the absence of external forcing the energy in the system decays according to the cooling law proposed by\textsuperscript{28}. Beyond this homogeneous cooling state (HCS) the system spontaneously develops clusters\textsuperscript{29,30}. The origin\textsuperscript{29,30}, growth and characteristic length scales of these clusters\textsuperscript{31} have been quite extensively researched\textsuperscript{32–36}. It is also recognized that granular gases do not exhibit the separation of scales observed in molecular gases\textsuperscript{37}. Here we analyze simulation results of clustering in granular gases to quantify fluctuations.

Molecular dynamics simulations of an inelastic hard–sphere gas comprising 117500 spheres at a reduced density $nd_p^3 = 0.157$ ($d_p$ is the particle diameter), are performed using an event–driven algorithm\textsuperscript{38} that is optimized using cell linked–lists. The periodic cubic domain is centered at the origin and has unit length ($\sim 90d_p$) in the three Cartesian directions. The mean free path $\lambda$ computed as $\lambda = 1/(\sqrt{2n\pi d_p^2}) = 1.43d_p$. Collisions are assumed to be binary and instantaneous, with the velocities of the collision partners updated according to

$$v'_{1,2} = v_{1,2} + \frac{(1+\varepsilon)}{2}[k \cdot v_{12}]k,$$

where $k$ is the unit vector along the line joining particle centers 1 and 2, $v_{12} = v_1 - v_2$ is the relative velocity, $v_{1,2}$ are the pre–collisional velocities with the superscript prime denoting the post–collisional velocities, and $\varepsilon$ is the normal coefficient of restitution. Statistically identical initial configurations of non-overlapping particles are generated according to a Matèrn hard–core point process as described in\textsuperscript{26}, using the same homogeneous number
density $n$ and pair-correlation $g(r)$ for each realization. Inelasticity is introduced only after each particle has undergone at least 100 elastic collisions. Results are reported in scaled time $\tau = t\nu(0)$, where

$$\nu(0) = (16/3)nd_p^2g(2d_p)(1 - \varepsilon^2)\sqrt{\pi T(0)/m}.$$ 

Figure 4 shows two snapshots of the particle positions, with a uniform distribution at initial time $\tau = \tau_0$ (panel 1), and significant cluster formation at $\tau = \tau_3 \approx 10^4$ (panel 2) that corresponds to a temporal regime beyond the HCS, where the system departs from Haff’s cooling law. Every 5th particle has been plotted for the sake of clarity.

FIG. 4. Snapshots of particle positions in a homogeneously cooling granular gas: a uniform distribution at initial time $\tau = \tau_0$ (panel 1), and significant cluster formation at $\tau = \tau_3 \approx 10^4$ (panel 2). Results are reported in scaled time $\tau = t\nu(0)$. Every 5th particle has been plotted for the sake of clarity.

The estimate for the SSFM in a measurement volume is computed by ensemble-averaging over 100 multiple independent simulations (MIS). The SSFM is estimated for a sequence of measurement volumes $V_m$ of increasing size, all centered at the origin. (Invariance of the SSFM with respect to shape of the averaging volume, and translation of its center, have been verified.) The departure from unity of the SSFM at initial $\tau = \tau_0$ and final time $\tau = \tau_3 \approx 10^4$ is shown as a function of the size of the measurement volume in Fig. 5. At initial time $\tau = 0$, the SSFM $\approx 1$ corresponding to spatially uncorrelated particle pairs for measurement volumes of all sizes. At $\tau = \tau_3$, the SSFM is different from unity for a separation length scale of at least $18.5d_p$. Clustering manifests itself in second–order statistics for this range of measurement volumes. For the smallest measurement volume the SSFM is significantly different from unity ($\sim 80\%$), indicating significant clustering. As the size of the measurement volume increases, the SSFM $\to 1$, indicating that particle pairs at large separations tend to become decorrelated.
FIG. 5. Departure from unity of the SSFM as a function of measurement volume size scaled by $L_{\text{ref}} = 1/n^{1/3}$ at initial time $\tau = \tau_0$, and after significant cluster formation at $\tau = \tau_3 \approx 10^4$. Results are reported in scaled time $\tau = t\nu(0)$, where $\nu(0) = 4nd_p^2\sqrt{k_B T(0)}$. during evolution of the inelastic granular gas. Also shown are 95\% confidence intervals based on the ensemble of 100 independent simulations.

These granular gas simulations tell us that fluctuations in number can be significant in granular and multiphase systems, and they may extend well into the mesoscale range of tens of particle diameters.

V. INFLUENCE OF FLUCTUATIONS ON CONSERVATION LAWS

We now explore the implications of this finding for first-order theories of multiphase flow such as the kinetic theory of gas-solid flow and the Eulerian two-fluid/multifluid theory.

A. Implications for kinetic theory of gas-solid flow

The transport equation for the one-particle distribution function in the kinetic theory of gas-solid flow reads:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x_k} [v_k f] + \frac{\partial}{\partial v_k} [\langle A_k | x, v ; t \rangle f] = \dot{f}_{\text{coll}},$$  \hspace{1cm} (4)
where the unclosed terms are: (i) $\langle A_k|\mathbf{x}, \mathbf{v} ; t \rangle$, which is the conditional acceleration of a particle due to hydrodynamic forces, and (ii) $\dot{f}_{\text{coll}}$, which is the source term due to collisions. The collisional source term is usually modeled using a Boltzmann or Enskog closure that writes this term as a product of one–particle distribution functions.

The presence of fluctuations in number over length scales extending tens of particle diameters requires a careful examination of existing closure assumptions in kinetic theory. Here we investigate what influence these fluctuations in number and volume have on particle–fluid interaction that affects the conditional acceleration of a particle due to hydrodynamic forces $\langle A_k|\mathbf{x}, \mathbf{v} ; t \rangle$.

B. Influence of clustering and fluctuations on particle–fluid interaction

We performed particle–resolved direct numerical simulation (DNS) of low Reynolds number flow past homogeneous assemblies of particles in fixed configurations that correspond to different pair–correlation functions (and hence, different SSFM) using the Particle–resolved Uncontaminated–fluid Reconcilable Immersed Boundary Method (PUReIBM) that has been developed at Iowa State University to simulate flow past fixed particle assemblies and freely evolving suspensions. It is shown elsewhere that PUReIBM is a numerically convergent and accurate particle-resolved DNS method for gas-solids flow.

We prepared different particle configurations, each with the same average number density corresponding to solid volume fractions of 0.1, 0.2 and 0.3, but with a different hard–core distance $h_c$. The pair correlation function $g(r)$ corresponding to different hard–core distances for volume fractions 0.1 and 0.3 are shown in Figs. 6(a) and 6(b) respectively. By varying $h_c$ we effectively control the length scale at which fluctuations in number manifest themselves through the pair correlation function. If $h_c$ equals the particle diameter, then the particles can get arbitrarily close until their surfaces touch, but with increasing $h_c$ the particles are forced to be farther away from each other. For random arrangements the average drag force exhibits a dependence on the hard–core distance normalized by particle diameter as shown in Fig. 7. For instance, for a solid volume fraction of 0.3, the average drag force increases by as much as 20% as the normalized hard–core distance increases from unity to 1.2. Based on these findings we conclude that it may not be possible to propose a closure for the conditional acceleration of a particle due to hydrodynamic forces $\langle A_k|\mathbf{x}, \mathbf{v} ; t \rangle$ that
FIG. 6. Pair correlation function corresponding to different hard–core distances for solid volume fractions of (a) 0.1 and (b) 0.3.

FIG. 7. Variation of average drag force with hard–core distance for solid solid volume fractions of 0.1, 0.2 and 0.3.

is dependent purely on first–order statistics. Since the kinetic theory does not contain a description of second–order statistics, this suggests it may be an inadequate level of closure for multiphase flows in which fluctuations of number and volume are significant.

Simulations by Kriebitzsch et al.\textsuperscript{44} also reveal the influence of neighbor particles on the drag force in Stokes flow. They looked at fluctuations in the drag force, whereas we examine the influence of neighbor particles on the mean drag force itself.

The importance of fluctuations in volume fraction and number raises an important ques-
tion regarding the dependence of drag force statistics on volume fraction fluctuations. In order to probe this question, we performed PUReIBM DNS of statistically homogeneous freely evolving gas–solid suspensions at a solid volume fraction of 0.2 and mean flow Reynolds number of 20. The instantaneous solid volume fraction and drag force in a measurement volume are computed and plotted in Fig. 8. We note that the fluctuations in volume fraction are important even in statistically homogeneous suspensions and these fluctuations give rise to considerable variation in the instantaneous drag force. Further, we note that the instantaneous drag force obtained at any volume fraction is very different from that predicted by a drag correlation. These results indicate that the drag force closures should incorporate the effect of the volume fraction fluctuations.

C. Implications for two–fluid theory

VI. FLUCTUATION HYDRODYNAMICS

As discussed earlier, a first–order description is not sufficient to characterize the phenomenon of clustering in multiphase flow. A clear connection between clustering mechanisms and their characterization is obtained by considering the transport equation for the density of the second factorial moment $\rho^{(2)}(r, w, t)$. This unnormalized two–particle density evolves by transport in pair–separation space $r$ and relative velocity space $w$ as follows:

$$
\frac{\partial}{\partial t} \rho^{(2)}(r, w, t) + \frac{\partial}{\partial r_k}[w_k \rho^{(2)}] + \frac{\partial}{\partial w_k} \left[ \left( \langle A_k^{(1)} \rangle - \langle A_k^{(2)} \rangle \right) \rho^{(2)} \right] = 0.
$$

(5)
This equation enables us to characterize clustering by considering the statistics of particle pairs and their evolution based on their relative velocity and relative acceleration \( \langle A_k^{(1)} \rangle - \langle A_k^{(2)} \rangle \). Recently Markutsya et al.\textsuperscript{45} have developed a coarse graining approach based on the transport equation for the two–particle density that has been successfully used to accurately predict nanoparticle aggregation using Brownian dynamics. Chun et al.\textsuperscript{46} have used the pair correlation function approach to characterize clustering of aerosol particles in isotropic turbulence. This approach can be extended to characterize the clustering phenomenon in granular and gas–solids flow.

VII. CONCLUSIONS

Fluctuations in number and volume occupied by dispersed–phase particles (or droplets/bubbles) in multiphase flow are characterized using a mathematical framework that relates these fluctuations to foundations of the kinetic theory of gas–solid flow and the two–fluid theory. It is shown that fluctuations in number and particle volume correspond to second moment of a random measure, whereas the kinetic theory and two–fluid theory are based on a first–order description. Therefore, neither of these theories is capable of representing the effect of fluctuations. Clustering is an important phenomenon in multiphase flow giving rise to fluctuations in number and volume fraction. In statistically homogeneous flow, clustering is characterized by the scaled second factorial moment, whose density is the pair correlation function. Hard–sphere molecular dynamics simulations of a cooling granular gas where particles interact in the absence of any ambient fluid through inelastic collisions reveal the formation of clusters that result in significant fluctuations in number and volume over length scales of \( \sim 20d_p \). Particle–resolved DNS of flow past homogeneous particle assemblies reveals that drag exhibits a dependence on the pair correlation (and the second factorial moment). Kinetic theory of gas–solid flow is based on the transport equation for the one–particle distribution function and employs a Boltzmann-Enskog closure is a first–order theory that cannot describe fluctuations. Since the particle–resolved DNS shows that the unclosed conditional acceleration term in the one–particle distribution function depends on second–order statistics, this suggests that kinetic theory with a Boltzmann-Enskog closure may be an inadequate level of closure for multiphase flow. The same conclusions also apply to the Eulerian two-fluid/multifluid theory because that is also a first–order theory.
REFERENCES


17 It is assumed that the system is ergodic and that the ensemble average converges to the mathematical expectation.


20 For identical particles the Klimontovich density is the same as the unscaled one-particle distribution.

21 For the simpler case of gas-solid flows we assume $N$ is not a function of $t$, but the general case of $N(t)$ is discussed by Subramaniam in the context of sprays.


26 D. Stoyan, W. S. Kendall, and J. Mecke, Stochastic geometry and its applications (J. Wiley and Sons, NY, 1995).


43. These configurations correspond to an ensemble where the total number of particles \(N\) is a constant.

